Exercise 49

(a) Evaluate the function $f(x) = x^2 - (2^x/1000)$ for $x = 1, 0.8, 0.6, 0.4, 0.2, 0.1$, and 0.05, and guess the value of

$$\lim_{x \to 0} \left(x^2 - \frac{2^x}{1000} \right)$$

(b) Evaluate $f(x)$ for $x = 0.04, 0.02, 0.01, 0.005, 0.003$, and 0.001. Guess again.

Solution

Evaluate $f(x)$ at several values of x to determine the limit.

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=1} = 0.998$$

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=0.8} = 0.638259$$

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=0.6} = 0.358484$$

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=0.4} = 0.158680$$

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=0.2} = 0.0388513$$

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=0.1} = 0.00892823$$

$$\left(x^2 - \frac{2^x}{1000} \right)\bigg|_{x=0.05} = 0.00146474$$

Therefore, one guess is

$$\lim_{x \to 0} \left(x^2 - \frac{2^x}{1000} \right) = 0.$$

Continue evaluating $f(x)$ for even smaller values of x.

$$\left(x^2 - \frac{2^x}{1000}\right)\Big|_{x=0.04} = 0.000571886$$

$$\left(x^2 - \frac{2^x}{1000}\right)\Big|_{x=0.02} = -0.000613959$$

$$\left(x^2 - \frac{2^x}{1000}\right)\Big|_{x=0.01} = -0.000906956$$

$$\left(x^2 - \frac{2^x}{1000}\right)\Big|_{x=0.005} = -0.000978472$$

$$\left(x^2 - \frac{2^x}{1000}\right)\Big|_{x=0.003} = -0.000993082$$

$$\left(x^2 - \frac{2^x}{1000}\right)\Big|_{x=0.001} = -0.000999693$$

Therefore, a second guess is

$$\lim_{x\to 0}\left(x^2 - \frac{2^x}{1000}\right) = -0.001.$$

The limit is actually

$$\lim_{x\to 0}\left(x^2 - \frac{2^x}{1000}\right) = 0 - \frac{2^0}{1000} = -\frac{1}{1000} = 0.001.$$

The graph below of $f(x)$ versus x confirms this.